3. PERIODIC PERCOLATION PROBLEMS.

3.1. Introduction of probability. Site vs bond problems.

Let $\mathcal{G}$ be a graph satisfying (2.1)-(2.5) with vertex set $\mathcal{V}$ and edge set $\mathcal{E}$. The most classical percolation model is the one in which all bonds of $\mathcal{G}$ are randomly assigned to one of two classes, all bonds being assigned independently of each other. This is called bond-percolation, and the two kinds of bonds are called the passable or open bonds and the blocked or closed bonds. Instead of partitioning the bonds one often partitions the sites into two classes. Again all sites are assigned to one class or the other independently of each other. One now speaks of site-percolation and uses occupied and vacant sites to denote the two kinds of sites. The crucial requirement in both models is the independence of the bonds or sites, respectively. This makes the states of the bonds or sites into a family of independent two-valued random variables. Accordingly the above models are called Bernoulli-percolation models.

Formally one describes the models as follows. One denotes the possible configurations of the bonds (sites) by $+1$ and -1 with $+1$ standing for passable (occupied) and -1 for blocked (vacant). The configuration space for the whole system is then

(3.1) $$\Omega_{\mathcal{E}} = \prod_{\mathcal{E}}\{-1,+1\} \quad \text{or} \quad \Omega_{\mathcal{V}} = \prod_{\mathcal{V}}\{-1,+1\}$$

A generic point of $\Omega_{\mathcal{E}}$ is denoted by $\omega = \{\omega(e)\}_{e \in \mathcal{E}}$ and for the σ-field $\mathcal{B}_{\mathcal{E}}$ in $\Omega_{\mathcal{E}}$ we take σ-field generated by the cylinder sets of $\Omega_{\mathcal{E}}$, i.e. the sets of the form

(3.2) $$\{\omega\colon \omega(e_1) = \varepsilon_1, \dots, \omega(e_n) = \varepsilon_n\}, \quad e_i \in \mathcal{E}, \quad \varepsilon_i = \pm 1.$$

For the probability measure on $\mathcal{B}_{\mathcal{E}}$ we choose a product measure

(3.3) $$P_{\mathcal{E}} = \prod_{e \in \mathcal{E}} \mu_e ,$$

where μ_e is defined by