2. WHICH GRAPHS DO WE CONSIDER?

This chapter discusses the graphs with which we shall work, as well as several graph-theoretical tools. Except for the basic definitions in Sect. 2.1-2.3 the reader should skip the remaining parts of this chapter until the need for them arises.

2.1 Periodic graphs.

Throughout this monograph we consider only graphs which are imbedded in $\mathbb{R}^d$ for some $d < \infty$. Only when strictly necessary shall we make a distinction between a graph and its image under the imbedding. Usually we denote the graph by $\mathcal{G}$, a generic vertex of $\mathcal{G}$ by u, v or w (with or without subscripts), and a generic edge of $\mathcal{G}$ by e, f or g (with or without subscripts). "Site" will be synonymous with "vertex", and "bond" will be synonymous with "edge". The collection of vertices of $\mathcal{G}$ will always be a countable subset of $\mathbb{R}^d$. The collection of edges of $\mathcal{G}$ will also be countable, and each edge will be a simple arc - that is, a homeomorphic image of the interval $[0,1]$ - in $\mathbb{R}^d$, with two vertices as endpoints but no vertices of $\mathcal{G}$ in its interior. In particular we take an edge to be closed, i.e., we include the endpoints in the edge. If e is an edge, then we denote its interior, i.e., e minus its endpoints, by $\mathring{e}$. We shall say that e is <u>incident to</u> v if v is an endpoint of e. We only allow graphs in which the endpoints of each edge are distinct; thus we assume

(2.1) $\qquad\qquad \mathcal{G}$ contains no loops.

We shall, however, allow several edges between the same pair of distinct vertices.

The notation

$$v_1 \mathcal{G} v_2 \text{ or equivalently } v_2 \mathcal{G} v_1$$

will be used to denote that v_1 and v_2 are <u>adjacent</u> or <u>neighbors</u> on $\mathcal{G}$. This means that there exists an edge of $\mathcal{G}$ with endpoints v_1 and v_2.