1. INTRODUCTION AND SUMMARY.

The earliest example of percolation was discussed in Broadbent (1954) and Broadbent and Hammersley (1957) as a model for the spread of fluid or gas through a random medium. The fluid, say, spreads through channels; fluid will move through a channel if and only if the channel is wide enough. There is therefore no randomness in the motion of the fluid itself, such as in a diffusion process, but only in the medium, i.e., in the system of channels. Broadbent and Hammersley modeled this as follows. The channels are the edges or bonds between adjacent sites on the integer lattice in the plane, $\mathbb{Z}^2$. Each bond is passable (blocked) with probability $p(q = 1 - p)$, and all bonds are independent of each other. Let P_p denote the corresponding probability measure for the total configuration of all the bonds. One is now interested in probabilistic properties of the configuration of passable bonds, and, especially in the dependence on the basic parameter p of these properties. Broadbent and Hammersley began with the question whether fluid from outside a large region, say outside $|x| < N$, can reach the origin. This is of course equivalent to asking for the probability of a passable path[1)] from the origin to $|x| \geq N$. For $v \in \mathbb{Z}^2$, let $W(v)$ be the union of all edges which belong to a passable path starting at v. This is the set of all points which can be reached by fluid from v. It is called the open component or cluster of v. $W(v)$ is empty iff the four edges incident on v are blocked. If we write W for $W(0)$, then the above question asks for the behavior for large N of

$$(1.1) \qquad P_p\{W \cap \{|x| \geq N\} \neq \emptyset\} .$$

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1) This is a path made up of passable edges between neighbors of $\mathbb{Z}^2$. Two successive edges of the path must have a vertex of $\mathbb{Z}^2$ in common. Precise definitions are given in later chapters.