CHAPTER 15

ADJOINTNESS AND QUANTIFIERS

"... *adjoints occur almost everywhere in many branches of Mathematics. ... a systematic use of all these adjunctions illuminates and clarifies these subjects.*"

Saunders Maclane

The isolation and explication of the notion of *adjointness* is perhaps the most profound contribution that category theory has made to the history of general mathematical ideas. In this final chapter we shall look at the nature of this concept, and demonstrate its ubiquity with a range of illustrations that encompass almost all concepts that we have discussed. We shall then see how it underlies the proof of the Fundamental Theorem of Topoi, and finally examine its role in a particular analysis of quantifiers in a topos.

15.1. Adjunctions

The basic data for an *adjoint situation*, or *adjunction*, comprise two categories, $\mathscr{C}$ and $\mathscr{D}$, and functors F and G between them

$$\mathscr{C} \underset{G}{\overset{F}{\rightleftarrows}} \mathscr{D}$$

in each direction, enabling an interchange of their objects and arrows. Given $\mathscr{C}$-object a and $\mathscr{D}$-object b we obtain

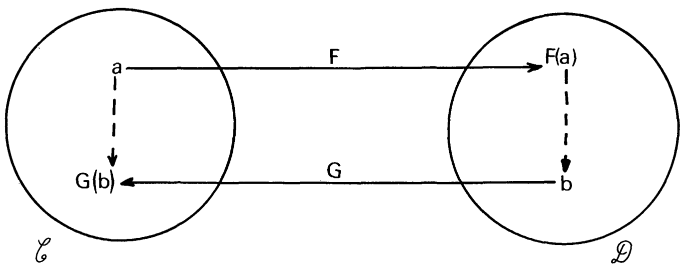

Fig. 15.1.