CHAPTER 11

ELEMENTARY TRUTH

> "... *a new theory, however special its range of application, is seldom or never just an increment to what is already known. Its assimilation requires the reconstruction of prior theory and the re-evaluation of prior fact, an intrinsically revolutionary process that is seldom completed by a single man and never overnight.*"
>
> Thomas Kuhn.

This chapter marks a change in emphasis towards an approach that will be more descriptive than rigorous. Our major concern will as usual be to analyse classical notions and define their categorial counterparts, but the detailed attention to verification of previous chapters will often be foregone. The proof that these generalisations work "as they should" will thus at times be left to the reader.

11.1. The idea of a first-order language

The propositional language PL of §6.3 is quite inadequate to the task of expressing the most basic discourse about mathematical structures. Take for example a structure $\langle A, R\rangle$ consisting of a binary relation R on a set A (i.e. $R \subseteq A \times A$). Let c be a particular element of A and consider the sentence "if every x is related by R to c, then there is some x to which c is related by R". If the "range" of the variable x is A, then this sentence is certainly true. For, if everything is related to c, then in particular c is related to c, so c is related to something. To see the structure of the sentence a little more clearly let

α abbreviate "for all x, xRc"