CHAPTER 8

INTUITIONISM AND ITS LOGIC

> *"Let those who come after me wonder why I built up these mental constructions and how they can be interpreted in some philosophy; I am content to build them in the conviction that in some way they will contribute to the clarification of human thought."*
>
> L. E. J. Brouwer

8.1. Constructivist philosophy

For a considerable period after the Calculus was discovered by Newton and Leibnitz in the late 17th century, there was controversy and disagreement over its fundamental concepts. Notions of infinitely small quantities, and limits of infinite sequences remained shrouded in mystery, and some of the statements made them look rather strange today (e.g. "A quantity that is increased or decreased infinitely little is neither increased nor decreased" (J. Bernoulli)). The subject acquired a rigorous footing in the 19th century, initially through the development by Cauchy of precise definitions of the concepts of limit and convergence. Later came the "arithmetisation of analysis" by Weierstrass and others, that produced a purely algebraic treatment of the real number system. A significant consequence of this was that analysis began to be separated from its grounding in physical intuition (cf. Weierstrass' proof of the existence of a (counter intuitive?) continuous nowhere-differentiable function). This, along with other factors like the development of non-Euclidean geometry, contributed to the recognition that mathematical structures have an abstract conceptual reality quite independently of the physical world.

Also important during this time was the work of Dedekind and Peano on the number systems. The real numbers were constructed from the rationals, the rationals from the integers, and the integers in turn from the