

TOPOS STRUCTURE: FIRST STEPS

“The development of elementary topoi by Lawvere and Tierney strikes this writer as the most important event in the history of categorical algebra since its creation . . . It is not just that they proved these things, its that they dared to believe them provable.”

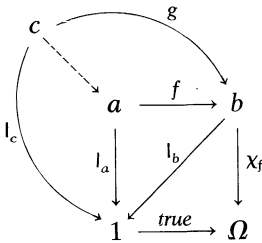
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5.1. Monics equalise

In §3.10 it was stated that an injective function $f: A \rightarrow B$ is an equaliser for a pair of functions g and h . We now see that g is $\chi_{\text{Im } f}: B \rightarrow 2$ and h is the composite of $!: B \rightarrow \{0\}$ and $\text{true}: \{0\} \rightarrow \{0, 1\}$. This situation generalises directly:—

THEOREM 1: *If $f: a \rightarrow b$ is a monic \mathcal{E} -arrow (\mathcal{E} any topos) then f is an equaliser of χ_f and $\text{true}_b = \text{true} \circ !_b$.*

PROOF: Since the pullback square of



commutes, and $l_a = l_b \circ f$, we have $\chi_f \circ f = \text{true}_b \circ f$. But if $\chi_f \circ g = \text{true}_b \circ g$

