CHAPTER 4

INTRODUCING TOPOI

> *"This is the development on the basis of elementary (first-order) axioms of a theory of "toposes" just good enough to be applicable not only to sheaf theory, algebraic spaces, global spectrum, etc. as originally envisaged by Grothendieck, Giraud, Verdier, and Hakim but also to Kripke semantics, abstract proof theory, and the Cohen–Scott–Solovay method for obtaining independence results in set theory."*
>
> F. W. Lawvere

4.1. Subobjects

If A is a subset of B, then the inclusion function $A \hookrightarrow B$ is injective, hence monic. On the other hand any monic function $f: C \rightarrowtail B$ determines a subset of B, viz $\operatorname{Im} f = \{f(x) \colon x \in C\}$. It is easy to see that f induces a bijection between C and $\operatorname{Im} f$, so $C \cong \operatorname{Im} f$.

Thus the domain of a monic function is isomorphic to a subset of the codomain. Up to isomorphism, the domain is a subset of the codomain. This leads us to the categorial versions of subsets, which are known as *subobjects*:

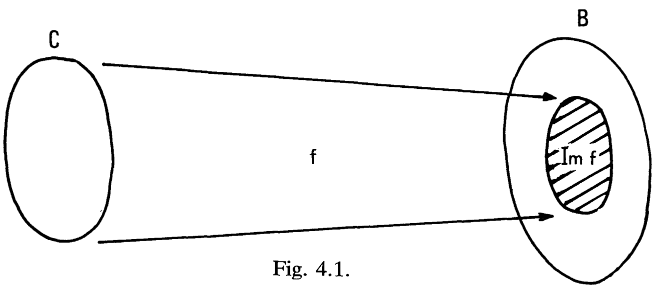

Fig. 4.1.