ARROWS INSTEAD OF EPSILON

"The world of ideas is not revealed to us in one stroke; we must both permanently and unceasingly recreate it in our consciousness".

René Thom

In this chapter we examine a number of standard set-theoretic constructions and reformulate them in the language of arrows. The general theme, as mentioned in the introduction, is that concepts defined by reference to the "internal" membership structure of a set are to be characterised "externally" by reference to connections with other sets, these connections being established by functions. The analysis will eventually lead us to the notions of *universal property* and *limit*, which encompass virtually all constructions within categories.

3.1. Monic arrows

A set function $f: A \to B$ is said to be *injective*, or *one-one* when no two distinct inputs give the same output, i.e. for inputs $x, y \in A$,

$$\text{if} \quad f(x) = f(y), \quad \text{then} \quad x = y.$$

Now let us take an injective $f: A \to B$ and two "parallel" functions $g, h: C \rightrightarrows A$ for which

$$\begin{array}{ccc} C & \xrightarrow{g} & A \\ {\scriptstyle h}\downarrow & & \downarrow{\scriptstyle f} \\ A & \xrightarrow{f} & B \end{array}$$

commutes, i.e. $f \circ g = f \circ h$.

Then for $x \in C$, we have $f \circ g(x) = f \circ h(x)$, i.e. $f(g(x)) = f(h(x))$. But as f is injective, this means that $g(x) = h(x)$. Hence g and h, giving the same