CHAPTER 2

WHAT CATEGORIES ARE

"... *understanding consists in reducing one type of reality to another.*"

Claude Levi-Strauss

2.1. Functions are sets?

A good illustration of the way in which set theory formalises an intuitive mathematical idea is provided by an examination of the notion of a *function.* A function is an association between objects, a correspondence that assigns to a given object one and only one other object. It may be thought of as a rule, or operation, which is applied to something to obtain its associated thing. A useful way of envisaging a function is as an input–output process, a kind of "black box" (see figure). For a given input the function produces a uniquely determined output. For example, the instruction "multiply by 6" determines a function which for input 2 gives output $6\times2=12$, which associates with the number 1 the number 6, which assigns 24 to 4, and so on. The inputs are called *arguments* of the function and the outputs *values,* or *images* of the inputs that they are produced by. If f denotes a function, and x an input, then the corresponding output, the image of x under f, is denoted $f(x)$. The above example may then be displayed as that function f given by the rule $f(x)=6x$.

If A is the set of all appropriate inputs to function f (in our example A will include the number 2, but not the Eiffel Tower), and B is a set that includes all the f-images of the members of A (and possibly the Eiffel Tower as well), then we say that f is a function *from A to B*. This is symbolised as $f:A\rightarrow B$ or $A\xrightarrow{f}B$. A is called the *domain* or *source* of f and B is the *codomain* or *target.*

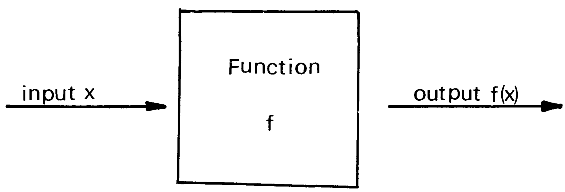

Fig. 2.1.