PROSPECTUS

> *". . . all sciences including the most evolved are characterised by a state of perpetual becoming."*
> Jean Piaget

The purpose of this book is to introduce the reader to the notion of a *topos*, and to explain what its implications are for logic and the foundations of mathematics.

The study of topoi arises within *category theory*, itself a relatively new branch of mathematical enquiry. One of the primary perspectives offered by category theory is that the concept of *arrow*, abstracted from that of *function* or *mapping*, may be used instead of the set membership relation as the basic building block for developing mathematical constructions, and expressing properties of mathematical entities. Instead of defining properties of a collection by reference to its members, i.e. *internal* structure, one can proceed by reference to its *external* relationships with other collections. The links between collections are provided by functions, and the axioms for a category derive from the properties of functions under composition.

A category may be thought of in the first instance as a universe for a particular kind of mathematical discourse. Such a universe is determined by specifying a certain kind of "object", and a certain kind of "arrow" that links different objects. Thus the study of topology takes place in a universe of discourse (category) with topological spaces as the objects and continuous functions as the arrows. Linear algebra is set in the category whose arrows are linear transformations between vector spaces (the objects); group theory in the category whose arrows are group homomorphisms; differential topology where the arrows are smooth maps of manifolds, and so on.

We may thus regard the broad mathematical spectrum as being blocked out into a number of 'subject matters' or categories (a useful way of lending coherence and unity to an ever proliferating and diversifying discipline). Category theory provides the language for dealing with these