Preface

Tracking back through the history, the vague shade of Gröbner bases already appeared in the famous paper[1] by F. S. Macaulay in 1927. The problem which Macaulay studied was to find a combinatorial characterization of Hilbert functions of residue class rings of the polynomial ring modulo homogeneous ideals. Macaulay succeeded in discovering the fundamental fact that the Hilbert function of the residue class ring of an arbitrary homogeneous ideal coincides with that of a certain monomial ideal. Macaulay's work had stimulated the algebraic study on enumerative combinatorics and promoted the birth of the historic area called "Commutative Algebra and Combinatorics," which originated in the work by Richard Stanley in 1975 on the upper bound conjecture for spheres by using the algebraic theory of Cohen–Macaulay rings.

The modern definition of Gröbner bases was independently introduced by Heisuke Hironaka in 1964 and Bruno Buchberger in 1965. Hironaka caught the idea of standard bases in the process of solving the outstanding problem, the resolution of singularities of algebraic varieties. On the other hand, Buchberger created the notion of Gröbner bases in his dissertation whose research topic had been given by his advisor Wolfgang Gröbner. Hironaka's standard bases work in the local ring, while Buchberger's Gröbner bases work in the polynomial ring. Apart from discussions of difference between the idea of standard bases and that of Gröbner bases, it turned out that Buchberger algorithm had opened the fascinating research area called "Computer Algebra."

After the pioneering work of Hironaka and Buchberger, however, for about twenty years, Gröbner bases had been out of the limelight. A turning point arose in the middle of 1980s, when David Bayer and Michael Stillman developed the computer software **Macaulay**, which has a great influence on computational aspects of commutative algebra and algebraic geometry. Since the theory of Gröbner bases was indispensable for developing **Macaulay**, Gröbner bases became common knowledge for researchers on commutative algebra and algebraic geometry.

The entry of Gröbner bases into the world of applied mathematics was achieved by Conti and Traverso in 1991, who proposed an algorithm to solve problems on integer programming by means of Gröbner bases of toric ideals. Toric ideals spread rapidly in the middle of 1990s. As one of the effective techniques to compute the dimension of the solution

[1] F. S. Macaulay, Some properties of enumeration in the theory of modular systems, *Proc. London Math. Soc.* **26** (1927), 531–555.