

## Foreword

When Takayuki Hibi invited me to the conference on Gröbner bases with the subtitle “The 50th Anniversary of Gröbner Bases,” I felt deeply pleased and grateful that the subject of my PhD thesis back in 1965 was receiving and is receiving so much attention.

In fact, the beginning in 1965 was quite hard and I had absolutely no feeling for what the impact of my work could be. In 1964, I was a working student of mathematics with a full-time job as a programmer at the computing center of the University of Innsbruck. I had thoroughly studied Van der Waerden’s and Gröbner’s book on commutative algebra (polynomial ideal theory) and I was determined (in spring 1964) to take the first problem Gröbner would present in his research seminar as my PhD thesis problem in order to finish my study as quickly as possible. By this, I hoped that my strenuous life as both a student and a programmer would soon be over. Luckily (in retrospect), the first problem Gröbner presented was the problem of constructing, by an algorithm, a linearly independent basis for the residue ring of a polynomial ring modulo a polynomial ideal.

Unfortunately (in my subjective feeling at that time), the problem turned out to be quite tough although its specification was so easy to understand. Thus, I still suffered through quite a long time of working, during the day, as a programmer and working, in my “free time” but often also while waiting for computer output, on Gröbner’s problem. The problem was basically always on my mind and I alternated between looking to examples, trying to invent and prove some lemmata and experimenting with the computer. I will always remember the big relief I felt when one day — I was riding on my bike to the university — I suddenly “saw” that the clue to the solution of the problem would be the consideration of what I then called the “ $S$ -polynomials.” From there it was a comparatively short step to come up with a proof of the  $S$ -polynomial theorem and, in parallel, with the  $S$ -polynomial algorithm for computing the special kind of ideal bases which I later called “Gröbner” bases. And, it was then clear that, as soon as one has a Gröbner basis for an ideal, the original problem of Gröbner and also a couple of other fundamental problems about polynomial ideals could be solved by algorithms.

In retrospect, what appeared for me to be a big tension between pure mathematics, theorems, structure, and proving on the one side and algorithms, data structures, computer hardware intricacies, programming languages and efficiency on the other hand, spanned the axes for my entire professional life.