

## Preface

The Seventh MSJ International Research Institute of the Mathematical Society of Japan was held in Tokyo for ten days from June 3rd to 12th, 1998. The theme was ‘Class Field Theory – its Centenary and Prospect’, which is taken as the title of the book. The program of this international conference is attached at the end of the preface. This volume is a collection of articles contributed by the speakers of the conference. All but a few of them are full scale papers. Some of them are expository on those subjects which are of central issues of algebraic number theory, and are prepared by the leading experts; they contain important and interesting problems with extensive references. Some of them are historical, and vividly explain how number theorists were motivated and exchanged their mathematical ideas.

In 1920 Takagi published the complete version of his class field theory as ‘Ueber eine Theorie des relativ Abel’schen Zahlkörpers’ in J. Coll. Sci. Tokyo, vol. 41. Chapter V of it is devoted to an affirmative solution to ‘Kronecker’s youth-dream’. This problem asks, roughly speaking, whether *all* abelian extensions of an imaginary quadratic number field could be obtained by the singular moduli and special values of elliptic functions which have complex multiplication by the elements of the quadratic field; and it was reformulated in a general framework by Hilbert as the twelfth problem of his celebrated 23 problems. There is another problem behind class field theory: that is, the principal ideal theorem which was finally proved by Furtwängler in 1930 based on Artin’s general reciprocity law. Artin established this significant result in his short paper, ‘Beweis des allgemeinen Reziprozitätsgesetzes’, in Abh. Math. Sem. Univ. Hamburg, vol. 5 (1927). Like the former problem, the origin of the latter goes back to Kronecker. We may say, however, that the Takagi-Artin class field theory has its direct origins in several papers by Weber and by Hilbert published in 1897–1898:

H. Weber, Ueber Zahlengruppen in algebraischen Körpern I, Math. Ann. 48 (1897), 433–473; II, 49 (1897), 83–100; III, 50 (1898), 1–26;

D. Hilbert, Die Theorie der algebraischen Zahlkörper, Jber. Deutschen Math.-Ver. 4 (1897), 175–546; Über die Theorie der relativ-Abelschen Zahlkörper, Nachr. Akad. Wiss. Göttingen 5 (1898), 377–399;

(cf. e.g. K. Miyake, The Establishment of the Takagi-Artin Class Field Theory, in *The Intersection of History and Mathematics* (ed. J. W.