Preface

> In mathematics, as in any scientific research, we find two tendencies present. On the one hand, the tendency toward *abstraction* seeks to crystallize the *logical* relations inherent in the maze of material that is being studied, and to correlate the material in a systematic and orderly manner. On the other hand, the tendency toward *intuitive understanding* fosters a more immediate grasp of the objects one studies, a live *rapport* with them, so to speak, which stresses the concrete meaning of their relations.
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> As to geometry, in particular, the abstract tendency has here led to the magnificent systematic theories of Algebraic Geometry, of Riemannian Geometry, and of Topology; these theories make extensive use of abstract reasoning and symbolic calculation in the sense of algebra. Notwithstanding this, it is still as true today as it ever was that intuitive understanding plays a major role in geometry. And such concrete intuition is of great value not only for the research worker, but also for anyone who wishes to study and appreciate the results of research in geometry.
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> — David Hilbert [**SE**: Hilbert]

These words, written in 1934 by David Hilbert, the "father of Formalism," are from the Preface to *Geometry and the Imagination*.

The formalisms of differential geometry are considered by many to be among the most complicated and inaccessible of all the formal systems in mathematics. It is probably fair to say that most mathematicians do not feel comfortable with their understanding of differential geometry. In addition, there is little agreement about which formalisms to use or how to describe them, with the result that the starting definitions, notations and analytic descriptions vary widely from text to text. What all of these different approaches have in common are underlying geometric intuitions of the basic notions such as straightness (geodesic), smooth, tangent, curvature, and parallel transport.

In this book we will study a foundation for differential geometry based not on analytic formalisms but rather on these underlying geometric intuitions. This foundation should be accessible to anyone with a flexible geometric imagination. It may then be possible that this foundation will serve as a common starting point for the various analytic formalizations. We will explore some of these analytical formalisms. In addition, this geometric foundation relates more directly with our actual experiences of curves and surfaces both in the physical world and in the context of computer graphics.

I invite the reader to explore the basic ideas of differential geometry. I am interested in conveying a different approach to mathematics, stimulating the reader to a broader and deeper experience of mathematics. Active participation with these ideas, including exploring and writing about them, will give the reader a broader context and experience, which is vital for anyone who wishes to understand differential geometry at a deeper level. More and more of the formal analytical aspects of differential geometry have now been mechanized, and this mechanization is widely available on personal computers, but the experience of meaning in differential geometry is still a human enterprise that is necessary for creative work.

I believe that mathematics is a natural and deep part of human experience and that experiences of meanings in mathematics are accessible to everyone. Much of mathematics is not accessible through formal approaches except to those with specialized learning. However, through the use of non-formal experience and geometric imagery, many levels of meaning in mathematics can be opened up in a way that more human beings can experience and find intellectually challenging and stimulating.

This text builds on a foundation of intuitive geometric ideas and then ties them into the formalisms of extrinsic and intrinsic differential geometry. The first chapter is an extensive collection of examples of surfaces which are discussed as much as can be done using elementary techniques and geometric