APPENDIX I.

ON ALGEBRAIC CURVES IN SPACE.

404. GIVEN an algebraic curve (C) in space, let a point O be found, not on the curve, such that the number of chords of the curve that pass through O is finite; let the curve be projected from O on to any arbitrary plane, into the plane curve (f), and referred to homogeneous coordinates ξ, η, τ in that plane, whose triangle of reference has such a position that the curve does not pass through the angular point η, and has no multiple points on the line $\tau=0$; let the curve (C) be referred to homogeneous coordinates ξ, η, ζ, τ of which the vertex ζ of the tetrahedron of reference is at O. Putting $x=\xi/\tau$, $y=\eta/\tau$, $z=\zeta/\tau$, it is sufficient to think of x, y, z as Cartesian coordinates, the point O being at infinity. Thus the plane curve (f) is such that y is not infinite for any finite value of x, and its equation is of the form $f(y, x)=y^m+A_1y^{m-1}+\dots\dots+A_m=0$, where $A_1, \dots, A_m$ are integral polynomials in x; the curve (C) is then of order m; we define its deficiency to be the deficiency of (f); to any point (x, y) of (f) corresponds in general only one point (x, y, z) of (C), and, on the curve (C), z is not infinite for any finite values of x, y.

Now let $f'(y)=\partial f(y, x)/\partial y$; let ϕ be an integral polynomial in x and y, so chosen that at every finite point of (f) at which $f'(y)=0$, say at $x=a$, $y=b$, the ratio $(x-a)\,\phi/f'(y)$ vanishes to the first order at least; let $\alpha=\Pi\,(x-a)$ contain a simple factor corresponding to every finite value of x for which $f'(y)=0$; let $y_1, \dots, y_m$ be the values of y which, on the curve (f), belong to a general value of x, so that to each pair (x, y_i) there belongs, on the curve (C), only one value of z; considering the summation

$$\sum_{i=1}^{m} \frac{(c-y_1)\dots\dots(c-y_m)}{c-y_i}\,\alpha\left[\frac{z\phi}{f'(y)}\right]_{y=y_i},$$

where c is an arbitrary quantity, we immediately prove, as in § 89, Chap. VI., that it has a value of the form

$$\alpha\,(c^{m-1}u_1+c^{m-2}u_2+\dots\dots+u_m),$$

where $u_1,\dots, u_m$ are integral polynomials in x; putting y_i for c, after division by α, we therefore infer that z can be represented in the form

$$z=\psi/\phi,$$

where ϕ, ψ are integral polynomials in x and y, whereof ϕ is arbitrary, save for the conditions for the fractions $(x-a)\,\phi/f'(y)$. This is Cayley's monoidal expression of a curve in space with the adjunction of the theorem, described by Cayley as the capital theorem of Halphen, relating to the arbitrariness of ϕ (Cayley, *Collect. Works*, Vol. v. 1892, p. 614).