CHAPTER XI.

THE HYPERELLIPTIC CASE OF RIEMANN'S THETA FUNCTIONS.

199.   WE have seen (Chap. V.) that the hyperelliptic case* is a special one, characterised by the existence of a rational function of the second order.   In virtue of this circumstance we are able to associate the theory with a simple algebraical relation, which we may take to be of the form

$$y^2 = 4 (x - a_1) \dots (x - a_p) (x - c_1) \dots (x - c_{p+1}).$$

We have seen moreover (Chap. X. § 185) that in the hyperelliptic case, when p is greater than 2, there are always even theta functions which vanish for zero values of the argument.   We may expect, therefore, that the investigation of the relations connecting the Riemann theta functions with the algebraical functions will be comparatively simple, and furnish interesting suggestions for the general case.   It is also the fact that the grouping of the characteristics of the theta functions, upon which much of the ultimate theory of these functions depends, has been built up directly from the hyperelliptic case.

It must be understood that the present chapter is mainly intended to illustrate the general theory.   For fuller information the reader is referred to the papers quoted in the chapter, and to the subsequent chapters of the present volume.

* For the subject-matter of this chapter, beside the memoirs of Rosenhain, Göpel, and Weierstrass, referred to in § 173, Chap. X., which deal with the hyperelliptic case, and general memoirs on the theta functions, the reader may consult, Prym, *Zur Theorie der Functionen in einer zweiblättrigen Fläche* (Zürich, 1866); Prym, *Neue Theorie der ultraellip. Funct.* (zweite Aus., Berlin, 1885); Schottky, *Abriss einer Theorie der Abel. Functionen von drei Variabeln* (Leipzig, 1880), pp. 147—162 ; Neumann, *Vorles. über Riem. Theorie* (Leipzig, 1884) ; Thomae, *Sammlung von Formeln welche bei Anwendung der . . Rosenhain'schen Functionen gebraucht werden* (Halle, 1876) ; Brioschi, *Ann. d. Mat.* t. x. (1880), and t. xiv. (1886) ; Thomae, *Crelle*, LXXI. (1870), p. 201 ; Krause, *Die Transformation der hyperellip. Funct. erster Ordnung* (Leipzig, 1886); Forsyth, " Memoir on the theta functions," *Phil. Trans.*, 1882 ; Forsyth, " On Abel's theorem," *Phil. Trans.*, 1883 ; Cayley, "Memoir on the . . theta functions," *Phil. Trans.*, 1880, and *Crelle*, Bd. 83, 84, 85, 87, 88; Bolza, *Göttinger Nachrichten* 1894, p. 268.   The addition equation is considered in a dissertation by Hancock, Berlin, 1894 (Bernstein).   For further references see the later chapters of this volume which deal with theta functions.