CHAPTER VII.

Coordination of simple elements. Transcendental uniform functions.

120. We have shewn in Chapter II. (§§ 18, 19, 20), that all the fundamental functions are obtainable from the normal elementary integral of the third kind. The actual expression of this integral for any given form of fundamental equation, is of course impracticable without precise conventions as to the form of the period loops, and for numerical results it may be more convenient to use an integral which is defined algebraically. Of such integrals we have given two forms, one expressed by the fundamental integral functions (Chap. IV. §§ 45, 46), the other expressed in the terms of the theory of plane curves (Chap. VI. § 92, Ex. ix.). In the present Chapter we shew how from the integral $P_{z,c}^{x,a}$, obtained in Chap. IV.*, to determine algebraically an integral $Q_{z,c}^{x,a}$ for which the equation $Q_{z,c}^{x,a} = Q_{x,a}^{z,c}$ has place; incidentally the character of $P_{z,c}^{x,a}$, as a function of z, becomes plain; and therefore also the character of the integral of the second kind, $E_z^{x,a}$, which was found in Chap. IV. (§§ 45, 47).

This determination arises in close connexion with the investigation of the algebraic expression of the rational function of x which was obtained in § 49 and denoted by $\psi(x, a; z, c_1, \dots c_p)$. It was there shewn that every rational function of x can be expressed in terms of this function. It is shewn in this Chapter that any uniform function whatever, which has a finite number of distinct infinities, which may be essential singularities, can be expressed by such a function.

Further, it is here shewn how to obtain an uniform function of x having only one zero, at which it vanishes to the first order, and one infinity; and that any uniform function can be expressed in factors by means of this function.

* For the integral of the third kind obtained in Chap. VI. the reader may compare Clebsch and Gordan, *Theorie der Abel. Functionen* (Leipzig, 1866), p. 117, and, for other important results, Noether, *Math. Annal.* XXXVII. (1890), pp. 442, 448; also Cayley, *Amer. Journal*, V. (1882), p. 173.