II. THE RESOLVENT

13. We shall follow, with some material deviations, König's exposition of Kronecker's method of solving equations by means of the resolvent. The equations are in general supposed to be non-homogeneous; and homogeneous equations are regarded as a particular case. Thus a homogeneous equation in n variables represents a cone of $n-1$ dimensions with its vertex at the origin. *Homogeneous co-ordinates are excluded.*

The problem is to find all the solutions of any given system of equations $F_1 = F_2 = \dots = F_k = 0$ in n unknowns $x_1, x_2, \dots, x_n$. The unknowns are supposed if necessary to have been subjected to a homogeneous linear substitution beforehand, the object being to make the equations and their solutions of a general character, and to prevent any inconvenient result happening (such as an equation or polynomial being irregular* in any of the variables) which could have been avoided by a linear substitution at the beginning. In theoretical reasoning *this preliminary homogeneous substitution is always to be understood*; but is seldom necessary in dealing with a particular example.

The solutions we shall seek are (i) those, if any, which exist for x_1 when $x_2, x_3, \dots, x_n$ have arbitrary values; (ii) those which exist for x_1, x_2, not included in (i), when $x_3, \dots, x_n$ have arbitrary values; (iii) those which exist for x_1, x_2, x_3, not included in (i) or (ii), when $x_4, \dots, x_n$ have arbitrary values; and so on. A set of solutions for $x_1, x_2, \dots, x_r$ when $x_{r+1}, \dots, x_n$ have arbitrary values is said to be of *rank* r, and the *spread* of the points whose coordinates are the solutions is of *rank* r and *dimensions* $n-r$. If there are solutions of rank r and no solutions of rank $< r$ the system of equations $F_1 = F_2 = \dots = F_k = 0$ and the module $(F_1, F_2, \dots, F_k)$ are both said to be of rank r.

14. The polynomials $F_1, F_2, \dots, F_k$, and also all their factors are regular in x_1. Hence their common factor D can be found by the ordinary process of finding the H.C.F. of $F_1, F_2, \dots, F_k$ treated as polynomials in a single variable x_1. If D does not involve the variables we take it to be 1. If it does involve the variables the solutions of $D = 0$ treated as an equation for x_1 give the first set of solutions of the equations $F_1 = F_2 = \dots = F_k = 0$ mentioned above.

* A polynomial of degree l is said to be regular or irregular in x_1 according as the term $x_1{}^l$ is present in it or not.