ALGEBRAIC INVARIANTS

PART I

ILLUSTRATIONS, GEOMETRICAL INTERPRETATIONS AND APPLICATIONS OF INVARIANTS AND COVARIANTS.

1. Illustrations from Plane Analytics. If x and y are the coördinates of a point in a plane referred to rectangular axes, while x' and y' are the coördinates of the same point referred to axes obtained by rotating the former axes counter-clockwise through an angle θ, then

$$T: \qquad x=x' \cos \theta-y' \sin \theta, \quad y=x' \sin \theta+y' \cos \theta.$$

Substituting these values into the linear function

$$l=ax+by+c,$$

we get $a'x'+b'y'+c$, where

$$a'=a \cos \theta+b \sin \theta, \quad b'=-a \sin \theta+b \cos \theta.$$

It follows that

$$a'^2+b'^2=a^2+b^2.$$

Accordingly, a^2+b^2 is called an *invariant* of l under every transformation of the type T.

Similarly, under the transformation T let

$$L=Ax+By+C=A'x'+B'y'+C,$$

so that

$$A'=A \cos \theta+B \sin \theta, \quad B'=-A \sin \theta+B \cos \theta.$$