

and those obtained by adding to each of them the four classes of the first column. In this way, the following table is obtained:

o	ab	$a'b'$	$ab + a'b'$
ab'	a	b'	$a + b'$
$a'b$	b	a'	$a' + b$
$ab' + a'b$	$a + b$	$a' + b'$	1

By construction, each class of this table is the sum of those at the head of its row and of its column, and, by the data of the problem, it is equal to each of those in the same column. Thus we have 64 different consequences for any equality in the universe of discourse of 2 letters. They comprise 16 identities (obtained by equating each class to itself) and 16 forms of the given equality, obtained by equating the classes which correspond in each row to the classes which are known to be equal to them, namely

$$\begin{aligned} 0 &= ab' + a'b, & ab &= a + b, & a'b' &= a' + b', & ab + a'b' &= 1 \\ a &= b, & b' &= a', & ab' &= a'b, & a + b' &= a' + b. \end{aligned}$$

Each of these 8 equalities counts for two, according as it is considered as a determination of one or the other of its members.

51. Table of Causes.—The same table may serve to represent all the causes of the same equality in accordance with the following theorem:

When the consequences of an equality $N = 0$ are expressed in the form of determinations of any class U , the causes of this equality are deduced from the consequences of the *opposite* equality, $N = 1$, put in the same form, by changing U to U' in one of the two members.

For we know that the consequences of the equality $N = 0$ have the form

$$U = (N' + X) U + NYU',$$

and that the causes of the same equality have the form

$$U = N'XU + (N + Y)U'.$$