39. Conditions of Impossibility and Indetermination.—The preceding theorem enables us to find the conditions under which an equation of several unknown quantities is impossible or indeterminate. Let $f(x, y, z \ldots)$ be the first member supposed to be developed, and $a, b, c \ldots, k$ its coefficients. The necessary and sufficient condition for the equation to be possible is

$$abc \ldots k = 0.$$

For, (1) if f vanishes for some value of the unknowns, its inferior limit $abc \ldots k$ must be zero; (2) if $abc \ldots k$ is zero, f may become equal to it, and therefore may vanish for certain values of the unknowns.

The necessary and sufficient condition for the equation to be indeterminate (identically verified) is

$$a + b + c \ldots + k = 0.$$

For, (1) if $a + b + c + \ldots + k$ is zero, since it is the superior limit of f, this function will always and necessarily be zero; (2) if f is zero for all values of the unknowns, $a + b + c + \ldots + k$ will be zero, for it is one of the values of f.

Summing up, therefore, we have the two equivalences

$$\sum [f(x, y, z, \ldots) = 0] = (abc \ldots k = 0).$$

$$\prod [f(x, y, z \ldots) = 0] = (a + b + c \ldots + k = 0).$$

The equality $abc \ldots k = 0$ is, as we know, the resultant of the elimination of all the unknowns; it is the consequence that can be derived from the equation (assumed to be verified) independently of all the unknowns.

40. Solution of Equations Containing Several Unknown Quantities.—On the other hand, let us see how we can solve an equation with respect to its various unknowns, and, to this end, we shall limit ourselves to the case of two unknowns

$$axy + bxy' + cx'y + dx'y' = 0.$$