

From this remark, PORETSKY concluded that, in general, the solution of an equation is neither a consequence nor a cause of the equation. It is a cause of it in the particular case in which

$$ab = 0,$$

and it is a consequence of it in the particular case in which

$$(a'b' = 0) = (a + b = 1).$$

But if  $ab$  is not equal to 0, the equation is unsolvable and the formula of solution absurd, which fact explains the preceding paradox. If we have at the same time

$$ab = 0 \quad \text{and} \quad a + b = 1,$$

the solution is both consequence and cause at the same time, that is to say, it is equivalent to the equation. For when  $a' = b$  the equation is determinate and has only the one solution

$$x = a' = b.$$

Thus, whenever an equation is solvable, its solution is one of its causes; and, in fact, the problem consists in finding a value of  $x$  which will verify it, *i. e.*, which is a cause of it.

To sum up, we have the following equivalence:

$$(ax + bx' = 0) = (ab = 0) \sum_{\text{u}} (x = a'u + bu')$$

which includes the following implications:

$$(ax + bx' = 0) < (ab = 0),$$

$$(ax + bx' = 0) < \sum_{\text{u}} (x = a'u + bu'),$$

$$(ab = 0) \sum_{\text{u}} (x = a'u + bu') < (ax + bx' = 0).$$

### 37. Elimination of Several Unknown Quantities.—

We shall now consider an equation involving several unknown quantities and suppose it reduced to the normal form, *i. e.*, its first member developed with respect to the unknown quantities, and its second member zero. Let us first concern ourselves with the problem of elimination. We can eliminate the unknown quantities either one by one or all at once.