33. Sums and Products of Functions.—It is desirable at this point to introduce a notation borrowed from mathematics, which is very useful in the algebra of logic. Let $f(x)$ be an expression containing one variable; suppose that the class of all the possible values of x is determined; then the class of all the values which the function $f(x)$ can assume in consequence will also be determined. Their sum will be represented by $\sum_x f(x)$ and their product by $\prod_x f(x)$. This is a new notation and not a new notion, for it is merely the idea of sum and product applied to the values of a function.

When the symbols $\sum$ and $\prod$ are applied to propositions, they assume an interesting significance:

$$\prod_x [f(x) = 0]$$

means that $f(x) = 0$ is true for *every* value of x; and

$$\sum_x [f(x) = 0]$$

that $f(x) = 0$ is true for *some* value of x. For, in order that a product may be equal to 1 (*i. e.*, be true), all its factors must be equal to 1 (*i. e.*, be true); but, in order that a sum may be equal to 1 (*i. e.*, be true), it is sufficient that only one of its summands be equal to 1 (*i. e.*, be true). Thus we have a means of expressing universal and particular propositions when they are applied to variables, especially those in the form: "For every value of x such and such a proposition is true", and "For some value of x, such and such a proposition is true", etc.

For instance, the equivalence

$$(a = b) = (ac = bc)\ (a + c = b + c)$$

is somewhat paradoxical because the second member contains a term (c) which does not appear in the first. This equivalence is independent of c, so that we can write it as follows, considering c as a variable x

$$\prod_x [(a = b) = (ax = bx)\ (a + x = b + x)],$$