

(that is to say, the part common to  $a'$  and  $x$ ). The solution is generally indeterminate (between the limits  $a'$  and  $b$ ); it is determinate only when the limits are equal,

$$a' = b,$$

for then

$$x = b + a'x = b + bx = b = a'.$$

Then the equation assumes the form

$$(ax + a'x' = 0) = (a' = x)$$

and is equivalent to the double inclusion

$$(a' < x < a') = (x = a').$$

**31. The Resultant of Elimination.**—When  $ab$  is not zero, the equation is impossible (always false), because it has a false consequence. It is for this reason that SCHRÖDER considers the resultant of the elimination as a *condition* of the equation. But we must not be misled by this equivocal word. The resultant of the elimination of  $x$  is not a *cause* of the equation, it is a *consequence* of it; it is not a *sufficient* but a *necessary* condition.

The same conclusion may be reached by observing that  $ab$  is the inferior limit of the function  $ax + bx'$ , and that consequently the function can not vanish unless this limit is 0.

$$(ab < ax + bx') (ax + bx' = 0) < (ab = 0).$$

We can express the resultant of elimination in other equivalent forms; for instance, if we write the equation in the form

$$(a + x') (b + x) = 0,$$

we observe that the resultant

$$ab = 0$$

is obtained simply by dropping the unknown quantity (by suppressing the terms  $x$  and  $x'$ ). Again the equation may be written:

$$a'x + b'x' = 1$$

and the resultant of elimination:

$$a' + b' = 1.$$