Demonstration.—First multiplying by x both members of the given equality [which is the first member of the entire secondary equality], we have

$$x = ax,$$

which, as we know, is equivalent to the inclusion

$$x < a.$$

Now multiplying both members by x', we have

$$0 = bx',$$

which, as we know, is equivalent to the inclusion

$$b < x.$$

Summing up, we have

$$(x = ax + bx') < (b < x < a).$$

Conversely,

$$(b < x < a) < (x = ax + bx').$$

For

$$(x < a) = (x = ax),$$
$$(b < x) = (bx' = 0).$$

Adding these two equalities member to member [the second members of the two larger equalities],

$$(x = ax)\ (0 = bx) < (x = ax + bx').$$

Therefore

$$(b < x < a) < (x = ax + bx'),$$

and thus the equivalence is proved.

30. Schröder's Theorem.[1]—The equality

$$ax + bx' = 0$$

signifies that x lies between a' and b.

Demonstration:

$$(ax + bx' = 0) = (ax = 0)\ (bx' = 0),$$
$$(ax = 0) = (x < a'),$$
$$(bx' = 0) = (b < x).$$

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[1] SCHRÖDER, *Operationskreis des Logikkalkuls* (1877), Theorem 20.