

For, if we develop with respect to x , we have

$$ax + bx' + abx + abx' = (a + ab)x + (b + ab)x' = ax + bx'.$$

Cor. 2. We have the equivalence

$$ax + bx' + c = (a + c)x + (b + c)x'.$$

For if we develop the term c with respect to x , we find

$$ax + bx' + cx + cx' = (a + c)x + (b + c)x'.$$

Thus, when a function contains terms (whose sum is represented by c) independent of x , we can always reduce it to the developed form $ax + bx'$ by adding c to the coefficients of both x and x' . Therefore we can always consider a function to be reduced to this form.

In practice, we perform the development by multiplying each term which does not contain a certain letter (x for instance) by $(x + x')$ and by developing the product according to the distributive law. Then, when desired, like terms may be reduced to a single term.

25. The Formulas of De Morgan.—*In any development of 1, the sum of a certain number of constituents is the negative of the sum of all the others.*

For, by hypothesis, the sum of these two sums is equal to 1, and their product is equal to 0, since the product of two different constituents is zero.

From this proposition may be deduced the formulas of DE MORGAN:

$$(a + b)' = a'b', \quad (ab)' = a' + b'.$$

Demonstration.—Let us develop the sum $(a + b)$:

$$a + b = ab + ab' + ab + a'b = ab + ab' + a'b.$$

Now the development of 1 with respect to a and b contains the three terms of this development plus a fourth term $a'b'$. This fourth term, therefore, is the negative of the sum of the other three.

We can demonstrate the second formula either by a correlative argument (*i. e.*, considering the development of 0 by factors) or by observing that the development of $(a' + b')$,