C. I.: The part common to any class whatever and to the null class is the null class; the sum of any class whatever and of the whole is the whole. The sum of the null class and of any class whatever is equal to the latter; the part common to the whole and any class whatever is equal to the latter.

P. I.: The simultaneous affirmation of any proposition whatever and of a false proposition is equivalent to the latter (i. e., it is false); while their alternative affirmation is equal to the former. The simultaneous affirmation of any proposition whatever and of a true proposition is equivalent to the former; while their alternative affirmation is equivalent to the latter (i. e., it is true).

Remark.—If we accept the four preceding formulas as axioms, because of the proof afforded by the double interpretation, we may deduce from them the paradoxical formulas

$$0 < x, \quad \text{and} \quad x < 1,$$

by means of the equivalences established above,

$$(a = ab) = (a < b) = (a + b = b).$$

14. The Law of Duality.—We have proved that a perfect symmetry exists between the formulas relating to multiplication and those relating to addition. We can pass from one class to the other by interchanging the signs of addition and multiplication, on condition that we also interchange the terms 0 and 1 and reverse the meaning of the sign $<$ (or transpose the two members of an inclusion). This symmetry, or *duality* as it is called, which exists in principles and definitions, must also exist in all the formulas deduced from them as long as no principle or definition is introduced which would overthrow them. Hence a true formula may be deduced from another true formula by transforming it by the principle of duality; that is, by following the rule given above. In its application the *law of duality* makes it possible to replace two demonstrations by one. It is well to note that this law is derived from the definitions of addition and multiplication (the formulas for which are reciprocal by duality)