2. $$(ab = a + b) < (a + b < ab),$$

(Comp.) $$(a + b < ab) = (a < ab)\ (b < ab),$$
$$(a < ab)\ (ab < a) = (a = ab) = (a < b),$$
$$(b < ab)\ (ab < b) = (b = ab) = (b < a).$$

Hence

$$(ab = a + b) < (a < b)\ (b < a) = (a = b).$$

12. **The Distributive Law.**—The principles previously stated make it possible to demonstrate the *converse distributive law,* both of multiplication with respect to addition, and of addition with respect to multiplication,

$$ac + bc < (a + b)c, \qquad ab + c < (a + c)\ (b + c).$$

Demonstration:

$$(a < a + b) < [ac < (a + b)c],$$
$$(b < a + b) < [bc < (a + b)c];$$

whence, by composition,

$$[ac < (a + b)c]\ [bc < (a + b)c] < [ac + bc < (a + b)c].$$

2. $$(ab < a) < (ab + c < a + c),$$
$$(ab < b) < (ab + c < b + c),$$

whence, by composition,

$$(ab + c < a + c)\ (ab + c < b + c) < [ab + c < (a + c)\ (b + c)].$$

But these principles are not sufficient to demonstrate the *direct distributive law*

$$(a + b)\,c < ac + bc, \qquad (a + c)\ (b + c) < ab + c,$$

and we are obliged to postulate one of these formulas or some simpler one from which they can be derived. For greater convenience we shall postulate the formula

(Ax. V). $$(a + b)\,c < ac + bc.$$

This, combined with the converse formula, produces the equality

$$(a + b)\,c = ac + bc,$$

which we shall call briefly the *distributive law.*

From this may be directly deduced the formula

$$(a + b)\ (c + d) = ac + bc + ad + bd,$$