Chapter 5

STRAIGHTNESS ON HYPERBOLIC PLANES

> [To son János:] For God's sake, please give it [work on hyperbolic geometry] up. Fear it no less than the sensual passion, because it, too, may take up all your time and deprive you of your health, peace of mind and happiness in life. — Wolfgang Bolyai (1775–1856) [**EM**: Davis and Hersh], page 220

We will now study some hyperbolic geometry. As with the cone and cylinder, we must use an intrinsic point of view on hyperbolic planes. This is especially true because, as we will see, there is no standard embedding of a complete hyperbolic plane into 3-space.

A SHORT HISTORY OF HYPERBOLIC GEOMETRY

Hyperbolic geometry initially grew out of the Building Structures Strand through the work of János Bolyai (1802–1860, Hungarian), and N. I. Lobachevsky (1792–1856, Russian). Hyperbolic geometry is special from a formal axiomatic point of view because it satisfies all the postulates (axioms) of Euclidean geometry except for the parallel postulate. In hyperbolic geometry straight lines can converge toward each other without intersecting (violating Euclid's Fifth Postulate), and there is more than one straight line through a point that does not intersect a given line (violating the usual high school parallel postulate, which states that through any point P not on a given line l there is one and only one line through P not intersecting l). See Figure 5.1.