Chapter 4

STRAIGHTNESS ON CYLINDERS AND CONES

If a cut were made through a cone parallel to its base, how should we conceive of the two opposing surfaces which the cut has produced — as equal or as unequal? If they are unequal, that would imply that a cone is composed of many breaks and protrusions like steps. On the other hand, if they are equal, that would imply that two adjacent intersection planes are equal, which would mean that the cone, being made up of equal rather than unequal circles, must have the same appearance as a cylinder; which is utterly absurd. — Democritus of Abdera (~460 – ~380 B.C.)

This quote shows that cylinders and cones were the subject of mathematical inquiry before Euclid (~365 – ~300 B.C.). In this chapter we investigate straightness on cones and cylinders. You should be comfortable with straightness as a *local intrinsic notion* — this is the bug's view. This notion of straightness is also the basis for the notion of *geodesics* in differential geometry. Chapters 4 and 5 can be covered in either order, but we think that the experience with cylinders and cones in Problem **4.1** will help the reader to understand the hyperbolic plane in Problem **5.1**. If the reader is comfortable with straightness as a local intrinsic notion, then it is also possible to skip Chapter 4 if Chapters 18 and 24 on geometric manifolds are not going to be covered. However, we suggest that you read the sections at the end of this chapter — Is "Shortest" Always "Straight"? and Relations to Differential Geometry — at least enough to find out what Euclid's Fourth Postulate has to do with cones and cylinders.