

## Chapter 3

# WHAT IS AN ANGLE?



A (plane) **angle** is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line. — Euclid, Elements, Definition 8

In this chapter you will be thinking about angles. In Problem **3.1** we will investigate various notions and definitions of angles and what it means for them to be considered to be the same (*congruent*). In Problem **3.2** we will prove the important *Vertical Angle Theorem* (VAT). It is not necessary to do these parts in order — you may find it easier to do Problem **3.2** before Problem **3.1** because it may help you think about angles. In a sense, you should be working on Problems **3.1** and **3.2** at the same time because they are so closely intertwined. This provides a valuable opportunity to apply and reflect on what you have learned about straightness in Chapters 1 and 2. This will also be helpful in the further study of straightness in Chapters 4 and 5; but, if you wish, you may study this chapter after Chapters 4 and 5.

### PROBLEM 3.1 WHAT IS AN ANGLE?

*Give some possible definitions of the term “angle.” Do all of these definitions apply to the plane as well as to spheres? What are the advantages and disadvantages of each? For each definition, what does it mean for two angles to be congruent? How can we check?*

### SUGGESTIONS

Etymologically, “angle” comes through Old English, Old French, Old German, Latin, and Greek words for “hook.” Textbooks usually give some variant of the definition: *An angle is the union of two rays (or segments) with a common endpoint.*