

APPENDIX A

MAKING MODELS

The Hyperbolic Plane from Paper Annuli

This is the paper-and-tape surface originally developed by William Thurston.¹ It may be constructed as follows.



Annular strips for making an annular hyperbolic plane

Cut out at least ten identical annular strips as in the photo. (An *annulus* is the region between two concentric circles, and we call an *annular strip* a portion of an annulus cut off by an angle from the center of the circles. Template provided.)

Attach the strips together by attaching the inner circle of one to the outer circle of the other or by attaching them end-to-end (i.e., along the straight ends). (When the straight ends of annular strips are attached together, you get annular strips with increasing “interior” angles, and eventually the angle will be greater than 360° .)



Taping annuli together.

The resulting surface is of course only an approximation of the desired surface. Let’s call the radius of the smaller circle r_1 and of the bigger circle r_2 , and their difference is $\rho = r_2 - r_1$. The actual hyperbolic plane is obtained by letting $\rho \rightarrow 0$ while holding the radius fixed. Note that since the surface is constructed the same everywhere (as $\rho \rightarrow 0$), it is homogeneous (that is, intrinsically and geometrically, every point has a

¹ William P. Thurston. *Three-Dimensional Geometry and Topology*, Vol. 1. Princeton, NJ: Princeton University Press, 1997; p. 50.