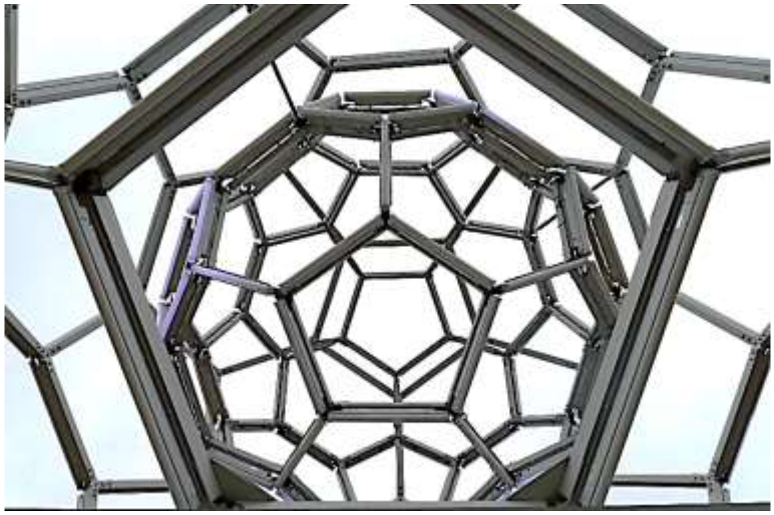
Chapter 23
POLYHEDRA



... if four equilateral triangles are put together, three of their plane angles meet to form a single solid angle,... When four such angles have been formed the result is the simplest solid figure ...

The second figure is composed of ... eight equilateral triangles, which yield a single solid angle from four planes. The formation of six such solid angles completes the second figure.

The third figure ... has twelve solid angles, each bounded by five equilateral triangles, and twenty faces, each of which is an equilateral triangle.

... Six squares fitted together complete eight solid angles, each composed by three plane right angles. The figure of the resulting body is the cube ...

There is remained a fifth construction, which the god used for arranging the constellations on the whole heaven. — Plato, Timaeus, 54e–55c [**AT**: Plato]

DEFINITIONS AND TERMINOLOGY

[The text in the brackets applies to polyhedra on a 3-sphere or a hyperbolic 3-space.] A ***tetrahedron***, $\Delta ABCD$, in 3-space [in a 3-sphere or a hyperbolic 3-space] is determined by any four points, A, B, C, D, called its ***vertices***, such that all four points do not lie on the same plane [great 2-sphere, great hemisphere] and no three of the points lie on the same line [geodesic]. The ***faces*** of the tetrahedron are the four [small] triangles ΔABC, ΔBCD, ΔCDA, ΔDAB. The ***edges*** of the tetrahedron are the six line [geodesic] segments AB, AC, AD, BC, BD, CD. The ***interior*** of the tetrahedron is the [smallest] 3-dimensional region that it bounds.

Tetrahedra are to three dimensions as triangles are to two dimensions. Every polyhedron can be dissected into tetrahedra, but the proofs are considerably more difficult than the ones from Problem **7.5b**, and in the discussion to Problem **7.5b** there is a polyhedron that is impossible to dissect into tetrahedra without adding extra vertices. There