

## Chapter 22

# 3-SPHERES AND HYPERBOLIC 3-SPACES



Let us, then, make a mental picture of our universe: ... as far as possible, a complete unity so that whatever comes into view, say the outer orb of the heavens, shall bring immediately with it the vision, on the one plane, of the sun and of all the stars with earth and sea and all living things as if exhibited upon a transparent globe. Bring this vision actually before your sight, so that there shall be in your mind the gleaming representation of a sphere, a picture holding all the things of the universe ... . Keep this sphere before you, and from it imagine another, a sphere stripped of magnitude and of spatial differences; cast out your inborn sense of Matter, taking care not merely to attenuate it: call on God, maker of the sphere whose image you now hold, and pray Him to enter. And may He come bringing His own Universe ... .

— Plotinus, *The Enneads*, V.8.9, Burdette, NY: Larson, 1992

In this chapter you will explore hyperbolic 3-space and the 3-dimensional sphere that extrinsically sits in 4-space. But intrinsically, if we zoom in on a point in a 3-sphere or a hyperbolic 3-space, then locally the experience of the space will become indistinguishable from an intrinsic and local experience of Euclidean 3-space. This is also our human experience in our physical universe. We will study these 3-dimensional spaces both because they are possible geometries for our physical universe and in order to see that these geometries are closely related to their 2-dimensional versions.

Try to imagine the possibility of our physical universe being a 3-sphere in 4-space. It is the same kind of imagination a 2-dimensional (2-D) being would need in order to imagine that it was on 2-sphere (ordinary sphere) in 3-space. In Problem **18.6** we thought about how a 2-D bug could determine (intrinsically) that it was on a 2-sphere. Now, we want to explore how the bug could imagine the 2-sphere in 3-space, that is, how could the bug imagine an extrinsic view of the 2-sphere in 3-space. In Problems **22.2** and **22.3** we will use linear algebra to help us talk about and analyze the 3-sphere in 4-space, but this will not solve the problem of imagining the 3-sphere in 4-space.