Chapter 19

GEOMETRIC SOLUTIONS OF QUADRATIC AND CUBIC EQUATIONS

Whoever thinks algebra is a trick in obtaining unknowns has thought it in vain. No attention should be paid to the fact that algebra and geometry are different in appearance. Algebras (jabbre and maqabeleh) are geometric facts which are proved by Propositions Five and Six of Book Two of [Euclid's] Elements.— Omar Khayyam, a paper [**AT:** Khayyam 1963]

In this chapter we will see how the results from Chapter 13 were used historically to solve equations. Quadratic equations were solved by "completing the square" — a real square. These results in turn lead to conic sections and cube roots and culminate in the beautiful general method from Omar Khayyam that can be used to find all the real roots of cubic equations. Along the way we shall see clearly some of the ancestral forms of our modern Cartesian coordinates and analytic geometry. We will point out several inaccuracies and misconceptions that have crept into the modern historical accounts of these matters. We urge you not to look at this only for its historical interest but also for the meaning it has in our present-day understanding of mathematics. This path is not through a dead museum or petrified forest; it passes through ideas that are very much alive and have something to say to our modern technological, increasingly numerical, world.