Chapter 17

PROJECTIONS (MODELS) OF HYPERBOLIC PLANES

In recent times the mathematical public has begun to occupy itself with some new concepts which seem to be destined, in the case they prevail, to profoundly change the entire order of classical geometry. — E. Beltrami (1868), when he developed the projective disk model (Problem **17.5**)

In this chapter we will study projections of a hyperbolic plane onto the plane and use these "models" to prove some results about the geometry of hyperbolic planes. In the case of hyperbolic planes, it is customary to call these "***models***" instead of "projections" because it was thought that there were no surfaces that were hyperbolic planes. As in the case of spherical projections, any projections (models) of the hyperbolic plane must distort some geometric properties; and with models it is more difficult to gain the intrinsic and intuitive experiences that are possible with the hyperbolic surfaces discussed in Chapter 5. Nevertheless, these models do give the most analytically accurate picture of hyperbolic planes and allow for more accurate and precise constructions and proofs. We take as our starting point the geodesic rectangular coordinates presented in Problem **5.2**. In order to connect these coordinates to the study of the models, we will need the results on circles from Chapter 14 and an analytic sophistication that is not necessary in other chapters in this book. However, no technical results from analysis are needed. The reader may bypass most of the analytic technicalities (which occur in Problems **17.1** and **17.2**) if the reader is willing to assume the results of Problem **17.2**, which make the connections between an annular hyperbolic plane and the upper-half-plane model and prove which curves in the