

Chapter 15

CIRCLES



... the Power of the World always works in circles, and everything tries to be round.

— Black Elk in *Black Elk Speaks* [GC: Neihardt]

Now we will study some important properties (Problem **15.1**) of circles in the plane that are stated and proved in Euclid's *Elements*. These planar results will be used in later chapters, and they are also often studied for their own interest. In Problem **15.2**, we will explore an extension of these results to circles on spheres (and later to hyperbolic planes). We will end the chapter with applications of these properties of circles (Problem **15.3**) to the ancient problem of trisecting angles (Problem **15.4**).

For Chapter 15, the only results needed from Chapters 9–14 are

PROBLEM 13.4a: The **AAA similarity criterion** for triangles on the (Euclidean) plane: *If two triangles are similar (have congruent angles), then the corresponding sides of the triangles are in the same proportion to one another.* [Needed throughout this and later chapters.]

PROBLEM 9.1: Side-Side-Side: *If two triangles (small triangles if on a sphere) have congruent corresponding sides, then the triangles are congruent.* [Needed throughout this and later chapters.]

PROBLEM 14.4: *Stereographic projection of a sphere onto a plane preserves angles, takes circles to circles (or to straight lines).* [Needed only for Problem **15.2**.]