

Chapter 13

SQUARE ROOTS, PYTHAGORAS, AND SIMILAR TRIANGLES



Juan Gris *Jar, Bottle, and Glass*, 1911

The diagonal of an oblong produces by itself both the areas which the two sides of the oblong produce separately. — Baudhayana, Sulbasutram, Sutra 48 [AT: Baudhayana]

This chapter is devoted to a subject that is not only of great theoretical interest but also of recreational interest, for dissection theory plays an important role in a rigorous development of area and volume and also furnishes a seemingly endless variety of attractive and challenging puzzles.

In the last chapter, we showed that two polygons on a sphere with the same area are equivalent by dissection because both are equivalent to the same biangle. In this chapter, we will prove an analogous result on the plane by showing that every planar polygon is equivalent by dissection to a square. This result is the basis for many popular dissection puzzles. Before reading this chapter, you should go through the Introduction to Dissection Theory and Problem **12.1** at the start of Chapter 12.

In the process of exploring this dissection theory, we will follow a path through a corner of the forest of mathematics — a path that has delighted and surprised the authors many times. We will bring with us the question, What are square roots? Along the way we will confront relationships between geometry and algebra of real numbers, in addition to similar triangles, the Pythagorean Theorem (the quote above is a statement of this theorem written before Pythagoras), and possibly the oldest written proof in geometry (at least 2600 years old). This path will lead to the solutions of quadratic and cubic equations in Chapter 19. We will let David's personal experience start us on this path.