Chapter 11

ISOMETRIES AND PATTERNS

What geometrician or arithmetician could fail to take pleasure in the symmetries, correspondences and principles of order observed in visible things? — Plotinus, The Enneads, II.9.16 [**AT:** Plotinus]

[Geometry is] the study of the properties of a space which are invariant under a given group of transformations. – F. Klein, Erlangen Program

Life forms illogical patterns. It is haphazard and full of beauties which I try to catch as they fly by, for who knows whether any of them will ever return? — Margot Fonteyn

Recall that in Chapter 1 we gave the following:

DEFINITION. An ***isometry*** is a transformation that preserves distances and angle measures.

In this chapter we will show (for the plane, spheres, and hyperbolic planes) that every isometry is the composition (product) of (not more than three) reflections, and we will determine all the different types of isometries. This finishes the study of reflections and rotations we started in Problem **5.4**. We will note the differences between the kinds of isometries that appear in the three geometries.

Then we will study patterns in these three spaces. Along the way we will look at some group theory through its origins, that is, geometrically.