Chapter 2
Extrinsic Curves

Introduction

The starting point of our extrinsic investigations is views of space. As is our ordinary experience of space, only a certain bounded region of space is within our field of view (f.o.v.) and within this field of view there are details too small to be distinguished. This makes sense in our experience of physical space, in computer graphics images, in fixed-point and floating point arithmetic, and also applies to the spaces of our imagination.

Borrowing from the idea of zooms in photography and computer graphics, when we want to investigate more detail of a figure we may zoom in on a point. Then less of the extent of space is included in our field of view but more details are now distinguishable.

We call a figure in our f.o.v. a point if it does not have two parts which are distinguishable from each other. [Note the connection between this notion and Euclid’s definition “A point is that which has no parts”.] We say that two figures in the f.o.v. are indistinguishable if each point of the first figure is indistinguishable from some point of the second figure, and each point of the second figure is indistinguishable from some point of the first figure.

For simplicity we shall assume that we see all parts of this field of view with equal clarity. (That is, we ignore the so-called peripheral vision, which is a region at the edge of our field of view where we can see less detail than in the center of the field of view.) Two distinguishable points in a field of view determine a line segment. In general we can subdivide this segment into 2 parts, 3 parts, 4 parts, etcetera, until each part becomes so small that it is indistinguishable from a point. We can quantify the tolerance of our vision in a field of view as the ratio \( \tau > 0 \) such that (see Figure 2.1):

- Every segment (in the field of view) is indistinguishable from a point if it has length less than \( \tau \rho \), where \( \rho \) is the radius of the f.o.v.
- Every segment (in the field of view) is distinguishable from a point if it has length greater than \( 2 \tau \rho \), where \( \rho \) is the radius of the f.o.v.

Figure 2.1. Tolerance \( \tau \) and radius \( \rho \) in a f.o.v.