Chapter 1

Surfaces and Straightness[†]

In the first six chapters of this book our study of differential geometry focuses on curves and surfaces. In later chapters we will see how to extend the results about surfaces to higher dimensional manifolds (the higher dimensional analogues of surfaces), especially to our physical universe which is a 3-dimensional (or 4-dimensional, if you include time) manifold.

In this chapter we will begin our study by examining a diverse collection of surfaces which will serve as examples throughout the remainder of the book. We will investigate each surface as much as we can without bringing in the differential notions of calculus. For each surface, starting with the plane, we will say what we can about what it means to be straight on the surface.

We begin with a question that encourages you to explore deeply a concept that is fundamental to all that will follow: We ask you to build a notion of straightness for yourself rather than accept a certain number of assumptions about straightness. Although difficult to formalize, straightness is a natural human concept.

PROBLEM 1.1. When Do You Call a Line Straight?

Look to your experiences. It might help to think about how you would explain straightness to a 5-year-old (or how the 5-year-old might explain it to you!). If you use a "ruler," how do you know if the ruler is straight? How can you check it? What properties do straight lines have that distinguish them from non-straight lines?

Think about the question in four related ways:

a. *How can you check in a practical way if something is straight—without assuming that you have a ruler, for then we will ask, "How can you check that the ruler is straight?"*

b. *How do you construct something straight—lay out fence posts in a straight line, or draw a straight line?*

c. *What symmetries does a straight line have? A symmetry of a geometric figure is a transformation (such as reflection, rotation, translation, or composition of them) which preserves the figure. For example, the letter* "T" *has reflection symmetry about a vertical line through its middle, and the letter* "Z" *has rotation symmetry if you rotate it half a revolution about its center.*

d. *Can you write a definition of "straight line"?*

[†] A small portion of this chapter is taken (somewhat revised) from the author's *Experiencing Geometry on Plane and Sphere* [**Tx**: Henderson]. It is used here with the permission of the publisher, Prentice-Hall, Inc.