**Problem 1.1. When Do You Call a Line Straight?**

Try to build for yourself a notion of a straight line. For example, think about how you would build a straight line and how you would check if the line you constructed is straight. Consider ways that you might convince someone that the line is, in fact, straight. Look to your experiences.

At first, you may look for examples of the physical world or natural straightness. You are likely to bring up ideas such as using a straight edge, stretching a string, sighting along a line, or, perhaps, following a laser beam. If so, then think about what is common among all of these “straight phenomena.”

As you look for properties of straight lines that distinguish them from non-straight lines, the following statement (which is often taken as a definition) is likely to arise: “A line is the shortest distance between two points.”

You may turn to this definition because it is, in fact, a common feature of the straightness expressed in the examples you have examined; perhaps, too, you are compelled to rely on it because this definition is most commonly used in high school mathematics classes. Whatever the case, the following questions may help you take a closer look at this definition:

♦ Can you always measure all the paths between two points?
♦ How do you find the shortest path?
♦ Is the shortest path between two points in fact a straight line?
♦ Is a straight line between two points always the shortest path?

You may cite examples of sighting along a line as a way to check for straightness. One example of “sighting” is a technique that is used for laying out fence posts. One can sight along a post on the end to see if any of the others are “out of line.” From the view at the end, all the posts should coincide. The notion of sighting along a line comes from a property of light; that is, light always travels along the path that takes the least time. This “least-time” path is straight whenever the light is traveling through a vacuum or a uniform medium such as air at a uniform pressure and temperature. When light crosses into a different medium, such as glass or water, it bends. This is the property that allows a lens to work. Ordinary light beams, even if narrowly constricted or focused, become diffused and fuzzy over a distance because of interference among the various wavelengths of light contained in an ordinary light beam. Laser beams overcome this problem because they consist of only a single wavelength of light which can be narrowly focused into a thin beam that does not degenerate over distances.

Discussions of these and other examples may lead you to a concept of “non-turning.” Here are some typical ideas you may present:

♦ “It is obvious that: $a$ is not the shortest path between $a$ and $b.$”