APPENDIX II.

ON MATRICES*.

405. A SET of n quantities

$$(x_1, \ldots, x_n)$$

is often denoted by a single letter x, which is then called a *row letter*, or a column letter. By the sum (or difference) of two such rows, of the same number of elements, is then meant the row whose elements are the sums (or differences) of the corresponding elements of the constituent rows. If m be a single quantity, the row letter mx denotes the row whose elements are $mx_1, \ldots, mx_n$. If x, y be rows, each of n quantities, the symbol xy denotes the quantity $x_1y_1 + \ldots\ldots + x_ny_n$.

406. The set of n equations denoted by

$$x_i = a_{i,1}\,\xi_1 + \ldots\ldots + a_{i,p}\,\xi_p, \qquad (i=1, \ldots\ldots, n)$$

where n may be greater or less than p, can be represented in the form $x = a\xi$, where a denotes a rectangular block of np quantities, consisting of n rows each of p quantities, the r-th quantity of the i-th row being $a_{i,r}$. Such a block of quantities is called a *matrix*; we call $a_{i,r}$ the (i, r)th element of the matrix. The sum (or difference) of two matrices, of the same number of rows and columns, is the matrix formed by adding (or subtracting) the corresponding elements of the component matrices. Two matrices are equal only when all their elements are equal; a matrix vanishes only when all its elements are zero. If $\xi_1, \ldots, \xi_p$ be expressible by m quantities $X_1, \ldots, X_m$ by the equations

$$\xi_r = b_{r,1}\,X_1 + \ldots\ldots + b_{r,m}\,X_m, \qquad (r=1, 2, \ldots\ldots, p),$$

so that $\xi = bX$, where b is a matrix of p rows and m columns, then we have

$$x_i = c_{i,1}\,X_1 + \ldots\ldots + c_{i,m}\,X_m, \qquad (i=1, \ldots\ldots, n),$$

or $x = cX$, where

$$c_{i,s} = a_{i,1}\,b_{1,s} + \ldots\ldots + a_{i,p}\,b_{p,s}, \qquad \begin{pmatrix} i=1, \ldots\ldots, n \\ s=1, \ldots\ldots, m \end{pmatrix},$$

* The literature of the theory of matrices, or, under a slightly different aspect, the theory of bilinear forms, is very wide. The following references may be given: Cayley, *Phil. Trans.* 1858, or *Collected Works*, vol. II. (1889), p. 475; Cayley, *Crelle*, L. (1855); Hermite, *Crelle*, XLVII. (1854); Christoffel, *Crelle*, LXIII. (1864) and LXVIII. (1868); Kronecker, *Crelle*, LXVIII. (1868) or *Gesam. Werke*, Bd. I. (1895), p. 143; Schläfli, *Crelle*, LXV. (1866); Hermite, *Crelle*, LXXVIII. (1874); Rosanes, *Crelle*, LXXX. (1875); Bachmann, *Crelle*, LXXVI. (1873); Kronecker, *Berl. Monatsber.*, 1874; Stickelberger, *Crelle*, LXXXVI. (1879); Frobenius, *Crelle*, LXXXIV. (1878), LXXXVI. (1879), LXXXVIII. (1880); H. J. S. Smith, *Phil. Trans.*, CLI. (1861), also, *Proc. Lond. Math. Soc.*, 1873, pp. 236, 241; Laguerre, *J. d. l'éc. Poly.*, t. XXV., cah. XLII. (1867), p. 215; Stickelberger, *Progr. poly. Schule, Zürich*, 1877; Weierstrass, *Berl. Monats.* 1858, 1868; Brioschi, *Liouville*, XIX. (1854); Jordan, *Compt. Rendus*, 1871, p. 787, and *Liouville*, 1874, p. 35; Darboux, *Liouville*, 1874, p. 347.