CHAPTER XIX.

On systems of periods and on general Jacobian functions.

343. The present chapter contains a brief account of some general ideas which it is desirable to have in mind in dealing with theta functions in general and more especially in dealing with the theory of transformation.

Starting with the theta functions it is possible to build up functions of p variables which have $2p$ sets of simultaneous periods—as for instance by forming quotients of integral polynomials of theta functions (Chap. XI., § 207), or by taking the second differential coefficients of the logarithm of a single theta function (Chap. XI., § 216, Chap. XVII., § 311 (δ)). Thereby is suggested, as a matter for enquiry, along with other questions belonging to the general theory of functions of several independent variables, the question whether every such multiply-periodic function can be expressed by means of theta functions*. Leaving aside this general theory, we consider in this chapter, in the barest outline, (i) the theory of the periods of an analytical multiply-periodic function, (ii) the expression of the most general single valued analytical integral function of which the second logarithmic differential coefficients are periodic functions.

344. If an uniform analytical function of p independent complex variables $u_1, \ldots, u_p$ be such that, for every set of values of $u_1, \ldots, u_p$, it is unaltered by the addition, respectively to $u_1, \ldots, u_p$, of the constants $P_1, \ldots, P_p$, then $P_1, \ldots, P_p$ are said to constitute a period column for the function. Such a column will be denoted by a single letter, P, and P_k will denote any one of $P_1, \ldots, P_p$. It is clear that if each of P, Q, $R, \ldots$ be period columns for the function, and λ, μ, ν, ... be any definite integers, independent of k, then the column of quantities $\lambda P_k + \mu Q_k + \nu R_k + \ldots$ is also a period column for the function; we shall denote this column by $\lambda P + \mu Q + \nu R + \ldots$, and say that it is a linear function of the columns P, Q, $R, \ldots$, the coefficients λ, μ, $\nu, \ldots$, in this case, but not necessarily

* Cf. Weierstrass, *Crelle*, LXXXIX. (1880), p. 8.