CHAPTER XVII.

THETA RELATIONS ASSOCIATED WITH CERTAIN GROUPS OF CHARACTERISTICS.

294. FOR the theta relations now to be considered*, the theory of the groups of characteristics upon which they are founded, is a necessary preliminary. This theory is therefore developed at some length. When the contrary is not expressly stated the characteristics considered in this chapter are half-integer characteristics†; a characteristic

$$\tfrac{1}{2}q = \tfrac{1}{2}\begin{pmatrix} q_1', & q_2', & \cdots, & q_p' \\ q_1, & q_2, & \cdots, & q_p \end{pmatrix}$$

is denoted by a single capital letter, say Q. The characteristic of which all the elements are zero is denoted simply by 0. If R denote another characteristic of half-integers, the symbol $Q + R$ denotes the characteristic, $S = \frac{1}{2}s$,

* The present chapter follows the papers of Frobenius, *Crelle*, LXXXIX. (1880), p. 185, *Crelle*, XCVI. (1884), p. 81. The case of characteristics consisting of n-th parts of integers is considered by Braunmühl, *Math. Annal.* XXXVII. (1890), p. 61 (and *Math. Annal.* XXXII. (1888), where the case $n=3$ is under consideration).

To the literature dealing with theta relations the following references may be given: Prym, *Untersuchungen über die Riemann'sche Thetaformel* (Leipzig, 1882); Prym u. Krazer, *Acta Math.* III. (1883); Krazer, *Math. Annal.* XXII. (1883); Prym u. Krazer, *Neue Grundlagen einer Theorie der allgemeinen Thetafunctionen* (Leipzig, 1892), where the method, explained in the previous chapter, of multiplying together the theta series, is fundamental: Noether, *Math. Annal.* XIV. (1879), XVI. (1880), where groups of half-integer characteristics are considered, the former paper dealing with the case $p=4$, the latter with any value of p; Caspary, *Crelle*, XCIV. (1883), XCVI. (1884), XCVII. (1884); Stahl, *Crelle*, LXXXVIII. (1879); Poincaré, Liouville, 1895; beside the books of Weber and Schottky, for the case $p=3$, already referred to (§§ 247, 199), and the book of Krause for the case $p=2$, referred to § 199, to which a bibliography is appended. References to the literature of the theory of the transformation of theta functions are given in chapter XX. In the papers of Schottky, in *Crelle*, CII. and onwards, and the papers of Frobenius, in *Crelle*, XCVII. and onwards, and in Humbert and Wirtinger (*loc. cit.* Ex. iv. p. 340), will be found many results of interest, directed to much larger generalizations; the reader may consult Weierstrass, *Berlin. Monatsber.*, Dec. 1869, and *Crelle*, LXXXIX. (1880), and subsequent chapters of the present volume.

† References are given throughout, in footnotes, to the case where the characteristics are n-th parts of integers. In these footnotes a capital letter, Q, denotes a characteristic whose elements are of the form q_i'/n, or of the form q_i/n, q_i', q_i being integers, which in the 'reduced' case are positive (or zero) and less than n. The abbreviations of the text are then immediately extended to this case, n replacing 2.