CHAPTER XII.

A PARTICULAR FORM OF FUNDAMENTAL SURFACE.

222. JACOBI's inversion theorem, and the resulting theta functions, with which we have been concerned in the three preceding chapters, may be regarded as introducing a method for the change of the independent variables upon which the fundamental algebraic equation, and the functions associated therewith, depend. The theta functions, once obtained, may be considered independently of the fundamental algebraic equation, and as introductory to the general theory of multiply-periodic functions of several variables; the theory is resumed from this point of view in chapter XV., and the reader who wishes may pass at once to that chapter. But there are several further matters of which it is proper to give some account here. The present chapter deals with a particular case of a theory which is historically a development* of the theory of this volume; it is shewn that on a surface which is in many ways simpler than a Riemann surface, functions can be constructed entirely analogous to the functions existing on a Riemann surface. The suggestion is that there exists a conformal representation of a Riemann surface upon such a surface as that here considered, which would then furnish an effective change of the independent variables of the Riemann surface. We do not however at present undertake the justification of that suggestion, nor do we assume any familiarity with the general theory referred to. The present particular case has the historical interest that in it a function has arisen, which we may call the Schottky-Klein prime function, which is of great importance for any Riemann surface.

223. Let α, β, γ, δ be any quantities whatever, whereof three are definitely assigned, and the fourth thence determined by the relation $\alpha\delta-\beta\gamma=1$. Let ζ, ζ' be two corresponding complex variables associated together by the relation $\zeta'=(\alpha\zeta+\beta)/(\gamma\zeta+\delta)$. This relation can be put into the form

$$\frac{\zeta'-B}{\zeta'-A}=\mu e^{i\kappa}\frac{\zeta-B}{\zeta-A},$$

* Referred to by Riemann himself, *Ges. Werke* (Leipzig, 1876), p. 413.