CHAPTER IX.

JACOBI'S INVERSION PROBLEM.

163. It is known what advance was made in the theory of elliptic functions by the adoption of the idea, of Abel and Jacobi, that the value of the integral of the first kind should be taken as independent variable, the variables, x and y, belonging to the upper limit of this integral being regarded as dependent. The question naturally arises whether it may not be equally advantageous, if possible, to introduce a similar change of independent variable in the higher cases. We have seen in the previous chapter that, if $u_1^{x,a}, \dots, u_p^{x,a}$ be any p linearly independent integrals of the first kind, the p equations

$$u_i^{x_1, a_1} + \dots\dots + u_i^{x_p, a_p} = -u_i^{x_{p+1}, a_{p+1}} - \dots\dots - u_i^{x_Q, a_Q}, \quad (i = 1, 2, \dots, p),$$

justify us in regarding the places $x_1, \dots, x_p$ as rationally determinable from the arbitrary places $a_1, \dots, a_Q, x_{p+1}, \dots, x_Q$; hence is suggested the problem, known as Jacobi's inversion problem*, which may be stated thus: *if $U_1, \dots, U_p$ be arbitrary quantities, regarded as variable, and $a_1, \dots, a_p$ be arbitrary fixed places, required to determine the nature and the expression of the dependence of the places $x_1, \dots, x_p$, which satisfy the p equations*

$$u_i^{x_1, a_1} + \dots\dots + u_i^{x_p, a_p} = U_i, \qquad (i = 1, 2, \dots, p),$$

upon the quantities $U_1, \dots, U_p$. It is understood that the path of integration from a_r to x_r is to be taken the same in each of the p equations, and is not restricted from crossing the period loops.

164. It is obvious first of all that if for any set of values $U_1, \dots, U_p$ there be one set of corresponding places $x_1, \dots, x_p$ of such general positions that no ϕ-polynomial (§ 101) vanishes in them, there cannot be another set of places, $x_1', \dots, x_p'$, belonging to the same values of $U_1, \dots, U_p$. For then we should have

$$u_i^{x_1', x_1} + \dots\dots + u_i^{x_p', x_p} = 0, \qquad (i = 1, 2, \dots, p),$$

* Jacobi, *Crelle* XIII. (1835), p. 55.