CHAPTER VI.

GEOMETRICAL INVESTIGATIONS.

80. IT has already been pointed out (§ 9) that the algebraical equation, associated with a Riemann surface, may be regarded as the equation of a plane curve; for the sake of distinctness we may call this curve the fundamental curve. The most general form of a rational function on the Riemann surface is a quotient of two expressions which are integral polynomials in the variables (x, y) in terms of which the equation associated with the surface is expressed. Either of these polynomials, equated to zero, may be regarded as representing a curve intersecting the fundamental curve. Thus we may expect that a comparison of the theory of rational functions on the Riemann surface with the theory of the intersection of a fundamental curve with other variable curves, will give greater clearness to both theories.

In the present chapter we shall make full use of the results obtainable from Riemann's theory and seek to deduce the geometrical results as consequences of that theory.

81. The converse order of development, though of more elementary character, requires much detailed preliminary investigation, if it is to be quite complete, especially in regard to the theory of the multiple points of curves. But the following account of this order of development may be given here with advantage (§§ 81—83). Let the term of highest aggregate degree in the equation of the fundamental curve $f(y, x) = 0$ be of degree n; and, in the usual way, regard the equation as having its most general form when it consists of all terms whose aggregate degree, in x and y, is not greater than n; this general form contains therefore $\frac{1}{2}(n+1)(n+2)$ terms. Suppose, further, that the curve has no multiple points other than ordinary double points and cusps, δ being the number of double points and κ of cusps. Consider now another curve, $\psi(x, y) = 0$, of order m, whose coefficients are at our disposal. By proper choice of these coefficients in ψ we can determine ψ to pass through any given points of f, whose number is not greater than the number of disposeable coefficients in ψ. Let k be the number of the prescribed points, and interpret the infinite intersections of f and ψ, in the usual way, so that their total number of intersections is mn. Then there