CHAPTER I.

THE SUBJECT OF INVESTIGATION.

1. THIS book is concerned with a particular development of the theory of the algebraic irrationality arising when a quantity y is defined in terms of a quantity x by means of an equation of the form

$$a_0y^n + a_1y^{n-1} + \ldots + a_{n-1}y + a_n = 0,$$

wherein $a_0, a_1, \ldots, a_n$ are rational integral polynomials in x. The equation is supposed to be irreducible; that is, the left-hand side cannot be written as the product of other expressions of the same rational form.

2. Of the various means by which this dependence may be represented, that invented by Riemann, the so-called Riemann surface, is throughout regarded as fundamental. Of this it is not necessary to give an account here*. But the sense in which we speak of a *place* of a Riemann surface must be explained. To a value of the independent variable x there will in general correspond n distinct values of the dependent variable y—represented by as many *places*, lying in distinct sheets of the surface. For some values of x two of these n values of y may happen to be equal: in that case the corresponding sheets of the surface may behave in one of two ways. Either they may just touch at one point without having any further connexion in the immediate neighbourhood of the point†: in which case we shall regard the point where the sheets touch as constituting two places, one in each sheet. Or the sheets may wind into one another: in which case we shall regard this winding point (or branch point) as constituting one place: this place belongs then indifferently to either sheet; the sheets here merge into one another. In the first case, if a be the value of x for which the sheets just touch, supposed for convenience of statement to be finite, and x a value

* For references see Chap. II. § 12, note.

† Such a point is called by Riemann "ein sich aufhebender Verzweigungspunkt": *Gesammelte Werke* (1876), p. 105.